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extended over the whole surface, where $d\sigma$ represents the element of area, and γ the angle which the radius vector makes with the external normal. [Goursat-Hedrick, *Analysis*, p. 325, ex. 9.]

SOLUTION BY S. W. REAVES, University of Oklahoma.

We may take for element of volume dV the cone (or pyramid) having the pole for vertex and the element of area $d\sigma$ for base. The altitude of this cone is clearly $\rho \cos \gamma$, and hence its volume is $dV = \frac{1}{3}\rho \cos \gamma d\sigma$. Hence the volume V, which is the sum of all the elements of volume, is, by the theory of definite integrals,

$$V = \tfrac{1}{3}\int\!\!\int \rho \cos \gamma d\sigma,$$

where the integration is to be taken over the entire surface.

344. Proposed by B. F. FINKEL, Drury College.

Solve the differential equation,

$$\frac{d^2y}{dx^2}(a^2 - x^2 - y^2) = \left(x\frac{dy}{ds} - y\frac{dx}{ds}\right)\left[1 + \left(\frac{dy}{dx}\right)^2\right]^{\frac{3}{2}}.$$

SOLUTION BY A. M. HARDING, University of Arkansas.

The equation may be written in the form

$$a^2 - x^2 - y^2 = \left(x\frac{dy}{ds} - y\frac{dx}{ds}\right)\frac{\left[1 + \left(\frac{dy}{dx}\right)^2\right]^{\frac{3}{2}}}{\frac{d^2y}{dx^2}} = \left(x\frac{dy}{ds} - y\frac{dx}{ds}\right)\rho,$$

where ρ is the radius of curvature at any point of the curve whose equation is sought.

Change to polar coördinates. Let ϕ be the angle the tangent makes with the initial line, and ψ the angle between the radius vector and the tangent, *i. e.*, $\phi = \theta + \psi$. Then

$$\frac{dr}{d\theta} = r \cot \psi$$

and

$$\rho = \frac{\frac{ds}{d\theta}}{\frac{d\phi}{d\theta}} = \frac{\sqrt{r^2 + \left(\frac{dr}{d\theta}\right)^2}}{1 + \frac{d\psi}{d\theta}} = \frac{\sqrt{r^2 + \left(\frac{dr}{d\theta}\right)^2}}{1 + \frac{d\psi}{dr}\cdot\frac{dr}{d\theta}} = \frac{r \csc \psi}{1 + r \cot \psi \frac{d\psi}{dr}}.$$

Also

$$\frac{dy}{ds} = \sin \phi, \qquad \text{and} \qquad \frac{dx}{ds} = \cos \phi.$$

Substituting, we obtain

$$a^2 - r^2 = r(\cos\theta\sin\phi - \sin\theta\cos\phi)\cdot\frac{r\csc\psi}{1 + r\cot\psi\dfrac{d\psi}{dr}}$$

$$= r\sin\psi\frac{r\csc\psi}{1 + r\cot\psi\dfrac{d\psi}{dr}} = \frac{r^2}{1 + r\cot\psi\dfrac{d\psi}{dr}},$$

whence

$$\cot\psi d\psi = \frac{2r^2 - a^2}{r(a^2 - r^2)}dr$$

or

$$\log\sin\psi = \tfrac{1}{2}\log\frac{1}{r^2(a^2 - r^2)} + \tfrac{1}{2}\log c,$$

i. e.,

$$\sin^2\psi = \frac{c}{r^2(a^2 - r^2)}.$$

But

$$\sin^2\psi = \frac{r^2}{r^2 + \left(\dfrac{dr}{d\theta}\right)^2}.$$

Therefore

$$\frac{c}{r^2(a^2 - r^2)} = \frac{r^2}{r^2 + \left(\dfrac{dr}{d\theta}\right)^2},$$

whence

$$\frac{\sqrt{c}dr}{\sqrt{-cr^2 + a^2r^4 - r^6}} = d\theta.$$

Let $r^2 = x$ and obtain

$$\frac{\tfrac{1}{2}\sqrt{c}dx}{x\sqrt{-c + a^2x - x^2}} = d\theta.$$

Integrate and obtain

$$\tfrac{1}{2}\sin^{-1}\frac{a^2r^2 - 2c}{r^2\sqrt{a^4 - 4c}} = \theta + \left(\frac{\pi}{4} - \beta\right),$$

after putting r^2 for x. Whence

$$\frac{a^2r^2 - 2c}{r^2\sqrt{a^4 - 4c}} = \cos(2\theta - 2\beta) = \cos 2\theta\cos 2\beta + \sin 2\theta\sin 2\beta.$$

Change to rectangular coördinates

$$[a^2 - \sqrt{a^4 - 4c}\cos 2\beta]x^2 - 2\sin 2\beta\sqrt{a^4 - 4c}\,xy + [a^2 + \sqrt{a^4 - 4c}\cos 2\beta]y^2 = 2c.$$

Now introduce a new constant $k^4 = a^4 - 4c$. The equation may then be written

$$[a^2 - k^2 \cos 2\beta]x^2 - 2k^2 \sin 2\beta \cdot xy + [a^2 + k^2 \cos 2\beta]y^2 = \frac{a^4 - k^4}{2}.$$

This is the equation of a system of conics, where the distance from the center to the focus is k, and where β is the angle between the major axis and the x-axis.

Also solved by ELIJAH SWIFT, who obtained the result $\phi = \int \frac{dr}{r\sqrt{cr^2(r^2 - a) - 1}}$, and by C. N. SCHMALL and who obtained $xy = c$.

The above solution by Professor HARDING is correct.

CALCULUS.

345. Proposed by C. N. SCHMALL, New York City.

Of all quadrilaterals which can be formed from four given sides, that which is inscriptible in a circle has the greatest area. [GOURSAT-HEDRICK, page 133, example 5.]

Proposer's Remark. Also show that when only three sides are known the length of the fourth side of the maximum quadrilateral is the root of a cubic equation.

SOLUTION BY THE PROPOSER.

Let the area $= u = \Delta ABC + \Delta ADC$. That is,

$$u = \tfrac{1}{2}(ab \sin \phi + cd \sin \psi). \tag{1}$$

Also

$$\overline{AC}^2 = a^2 + b^2 - 2ab \cos \phi$$

$$= c^2 + d^2 - 2cd \cos \psi.$$

Hence

$$a^2 + b^2 - 2ab \cos \phi = c^2 + d^2 - 2cd \cos \psi. \tag{2}$$

Now for a maximum we have from (1)

$$\frac{\partial u}{\partial \phi} = \frac{1}{2}\left(ab \cos \phi + cd \cos \psi \frac{\partial \psi}{\partial \phi}\right) = 0, \tag{3}$$

and from (2)

$$0 = 2ab \sin \phi - 2cd \sin \psi \frac{\partial \psi}{\partial \phi}. \tag{4}$$

Equating the values of $\partial\psi/\partial\phi$ obtained from (3) and (4), and reducing, we get

$$\frac{\cos \phi}{\sin \phi} = -\frac{\cos \psi}{\sin \psi}.$$